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Low Complexity Beamformers for Multiple Transmit and Receive Antennas

TECHNICAL FIELD

The field of the invention is telecommunications, in particular antenna systems for cellular systems.

BACKGROUND OF THE INVENTION

Various beamforming schemes for wireless systems equipped with multiple transmit and multiple receive antennas are known in the art. At present, space time coding schemes are currently proposed for multiple antenna systems.

Figure 1, taken from US Patent 6,584,302, assigned to the assignee hereof, presents a transceiver that has a group of antenna elements 200-204 for transmitting and receiving. The transceiver is usually a base station but it can also be a subscriber equipment. The antenna with several elements can be an antenna array or some other kind of cluster of antenna elements. Referring to receiver 260, each signal received from each antenna 200-204 enters RF-means 206-210 that convert the radio frequency signal to baseband signals in a known manner. The signals are digitized in A/D-converters 212-216. The digital baseband signals are multiplied by coefficients W_1 -W_M that form the shape of the beam of the antenna in multipliers 218-222. The coefficients W_1 -W_M are digital complex numbers. The receiver searches for the values of the coefficients W_1 -W_M that produce the best reception. Antenna responses are calculated in an antenna response unit 224 for each antenna element. The antenna responses are ranked and a subset of the set of the antenna responses is selected in rank and select unit 226.

A response of an antenna element is similar to an impulse response and is calculated

by using correlation. In the correlation a known pseudo-random spreading code is correlated with the received signal L times. L is the number of paths of the multipath propagated signal. After calculating one correlation value the spreading code is shifted by a time difference ΔT, which can be the same as the duration of a chip.

In the transmitter 262 the subset comprising at least one antenna response is fed to a coefficient unit 230 that calculates the coefficients a_1 -a_M for each antenna element 200-204 transmitting a signal. The signal to be transmitted is multiplied by the coefficients using the multipliers 232-236. The signal weighted by the coefficients a_1 -a_M is then converted to an analog signal by D/A-converters 238-242. After that, the analog signals are converted to radio frequency signals in RF-means 244-248 and the radio frequency signals are transmitted by the antenna elements 200-204.

Low receiver complexity is one of the important design goals for downlink transmission where the handset (receiver) is constrained in its computational abilities.

It is well known that channel state information at the transmitter can enhance the system performance significantly. However, in practical systems, only partial channel information may be available at the transmitter due to the limited nature of the feedback resources

Hence, it is important to design feedback based transmission schemes for the cases where partial channel information is available at the transmitter. Transmission schemes for single receive antenna systems utilizing quantized channel information have been developed, but are not satisfactory.

SUMMARY OF THE INVENTION

The invention relates to a design criterion and beamformer constructions, which make use of finite rate feedback in the system.

A feature of the invention is a beamforming scheme for wireless systems equipped with multiple transmit and multiple receive antennas.

Another feature of the invention is the use of mathematical formalism originally developed for unitary space-time constellations for a beamformer.

Another feature of the invention is the application of a beamformer construction that is equivalent to a spatial water-filling solution.

BRIEF DESCRIPTION OF THE DRAWING

Figure 1 illustrates in block diagram form a multiple antenna system according to the prior art.

Figures 2 - 6 illustrate performance of various configurations.

BEST MODE OF CARRYING OUT THE INVENTION

The beamforming schemes presented in this disclosure result in improved performance at provably lower computational complexity compared to the space time coding schemes currently proposed for multiple antenna systems. Low receiver complexity is one of the important design goals for downlink transmission where the handset (receiver) is constrained in its computational abilities.

It is well known that channel state information at the transmitter can enhance the system performance significantly. However, in practical systems, only partial channel information may be available at the transmitter due to the limited nature of the feedback resources. Hence, it is important to design feedback based transmission

schemes for the cases where partial channel information is available at the transmitter. Transmission schemes for single receive antenna systems utilizing quantized channel information have been developed.

In this work, we present a design criterion and beamformer constructions which make use of finite rate feedback in the system. In the first part of this disclosure, we present a unit rank beamforming strategy for multiple transmit and multiple receive antenna systems. In the second part, we present an algorithm to extend the beamformer codebook constructions to mimic a spatial water-filling solution with a finite number of feedback bits. We will show that both the schemes result in better performance at lower decoding complexity compared to space time coding. In particular, we can show that unit rank beamforming schemes are useful when the transmission rate is small. In fact, we can show that the unit rank beamforming schemes result in significant performance gains over space time coding schemes when $2^{R/r}/t < 1$, where R is the rate of transmission in bits/sec/Hz, r is the number of receive antennas and t is the number of transmit antennas. When this condition for unit rank beamforming is not met, i.e., for higher transmission rates, we propose higher rank beamforming schemes based on the spatial water-filling algorithm, using finite rate feedback.

Consider a multiple antenna system with t transmit antennas and r receive antennas, such as that illustrated in Figure 1. Let us suppose that we wish to transmit at a spectral efficiency of R bits/sec/Hz. We denote the t x 1 transmitted vector by X while the r x 1 received vector is denoted by Y. The additive white noise vector is denoted by N while the frequency non-selective Rayleigh fading channel between the transmit antennas and the receive antennas is given by the r x t matrix H. With this notation, the received signal Y can be expressed as

$$Y=HX+N. \tag{1}$$

The channel fading is assumed to be quasi-static over time; i.e., the channel remains constant within a frame while the channel realization is independent from frame to frame. We assume that the channel is known perfectly at the receiver. In practice, good channel estimates can be obtained at the receiver by employing preamble based training in the system. We also assume the existence of an error-free feedback channel from the receiver to the transmitter which carries B bits every frame. For simplicity, power adjustment over time; (i.e., temporal power control) is not performed.

We will first discuss unit rank beamforming schemes and analyze their performance. Unit rank beamforming schemes are optimal in the sense of minimizing outage probability in the important case when the number of receive antennas is restricted to 1. Further, we have shown that unit rank beamforming with multiple receive antennas is optimal in the sense of minimizing the pair-wise codeword error probability. Additionally, unit rank beamforming schemes result in simple decoding structures with low computational complexity.

We have shown that transmission along the dominant eigenvector of the channel minimizes the pairwise codeword error probability in the system. It has also been shown that the transmission along the dominant eigenvector of the channel maximizes received SNR while resulting in maximum diversity. We refer to this transmission strategy as the unit rank beamforming scheme.

It is an advantageous feature of the invention that the decoding complexity of the unit rank beamforming scheme is independent of the number of transmit antennas. Since there is a single stream of data (corresponding to the eigen channel with the best eigen value) the resulting encoder is a scalar encoder and hence independent of the number of transmit antennas. As a result, the corresponding decoder is also a scalar decoder and hence the decoding complexity is independent of the number of the transmit antennas. In contrast, space time codes encode across all the transmit

antennas in a joint fashion, thus resulting in a vector encoder whose order is given by the number of transmit antennas. In such a case, the corresponding vector decoder's complexity is exponential in the number of transmit antennas.

Consider the example of a finite size beamformer codebook given by $C = \{C_1, C_2, - C_N\}$. We can show that the quantizer which minimizes the outage probability is given by

$$\min_{C_i \in C} \|HC_i^\dagger\|_2^2, \tag{2}$$

where $\|.\|_2$ represents the l_2 norm on $\mathbb{C}^t$. Hence, a given channel realization H will be mapped to the beamforming vector C_i which minimizes expression (2). It can also be shown that as N gets large, the quantization rule given by (2) leads to the dominant eigenvector of the channel. This follows from the Rayleigh quotient, which states that $\|HV\|_2$ is maximized when V is the dominant eigenvector of $H^\dagger H$.

We can further establish a lower bound on the outage performance of the unit rank beamforming scheme when the beamforming codebook size is constrained to $N=2^B$ vectors. In particular, for t transmit antennas and r=2 receive antennas, we can show that the outage probability of the system is bounded below as follows:

$$P_{out}(R,P) \geq 1 - N(1+\gamma_0)e^{-\gamma_0} + e^{-\gamma_1}\left(\sum_{k=0}^{2t-1} \frac{N(1+\gamma_0)(\gamma_1-\gamma_0)^k - \gamma_1^k}{k!}\right)$$

$$-Ne^{-\gamma_1}\gamma_0 \frac{(\gamma_1-\gamma_0)^{2t-1}}{(2t-1)!} \tag{3}$$

Where $\gamma_0 = \frac{2^R - 1}{P}$ and γ_1 is a function of N, t and γ_0.

Hence, with the above quantization rule, all the beamformer constructions which were known for a single receive antenna can be adapted for multiple receive antennas also. The design criterion for good beamformer codebooks in the case of single receive antenna is therefore given by

$$\min_{C} \max_{i,j:i\neq j} |\langle C_i, C_j \rangle|. \tag{4}$$

Under appropriate circumstances, the above design criterion is mathematically equivalent to the design criterion of unitary space time constellations for non-coherent constellations. Hence, all the constructions available for unitary constellation design can also be used for the beamformer design problem with the quantization metric given by (2).

Figure 2 shows the performance of the quantized unit rank beamformers with four transmit antennas and two receive antennas transmitting at R=2 bits/sec/Hz (curves 104 and 106). The performances of a space-time coding scheme which does not use any channel state information (curve 110) as well as the spatial water-filling solution which requires complete channel state information (curve 102) are also given for comparison. With 6 bits of feedback information (curve 106), we see a gain of over 2.5 dB for the unit rank beamforming scheme over the space time coding scheme at an outage performance of 10^{-2}. The gains increase further as the number of feedback bits is increased. Note that we can potentially gain up to 4dB over space time codes using unit rank beamforming schemes. Further, the gap between the unit rank beamforming scheme (curve 104) and the higher rank water-filling scheme (curve 102) is less than 0.4dB for this rate of transmission. The performance gains are in addition to significant reduction in complexity for the beamforming scheme over the space time coding as already explained. Further, the performance of dominant

eigenvector beamforming (curve 104) which is the limit of the unit rank beamforming as the number of beamforming vectors gets large is also given in Figure 2.

In the next section, we will present higher rank beamforming schemes (spatial water-filling) with finite rate feedback, which provide significant performance gains over space time codes as well as unit rank beamforming schemes when the transmission rate is increased (in particular, when $2^{R/r}/t < 1$, where R is the rate of transmission, r is the number of receive antennas and t is the number of transmit antennas).

We now propose an algorithm to extend the unit rank beamforming approach for multiple receive antennas to a quantized spatial water-filling approach for the case of two receive antennas. The algorithm can be easily extended to the case of more than two receive antennas. Next generation handsets are expected to be equipped with no more than two antennas, due to size and cost constraints. Hence, the case of two receive antennas is important for downlink transmission in cellular systems.

For the case of a spatial water-filling solution, the transmitter needs to possess information about the eigenvectors as well as the eigenvalues of $\boldsymbol{H}^{\dagger}\boldsymbol{H}$. Note that the knowledge of the relative value of the eigenvalues (e.g. ratio of the eigenvalues) will not suffice for the water-filling power allocation. The invention employs a quantizer solution in which the eigenvectors and the power allocation vector are quantized independently. This separation imposes certain structure on the quantizer design, which advantageously reduces the complexity of implementation of the quantizer in practice.

In the case of two receive antennas ($r = 2$), $\boldsymbol{H}^{\dagger}\boldsymbol{H}$ can at most have two non-zero eigenvalues. Hence, the knowledge at the transmitter should comprise these two eigenvectors (corresponding to non-zero eigenvalues) as well as the corresponding eigenvalues. The inventors have realized that significant savings in feedback

resources can be obtained if the power allocation is made at the receiver and the information about the power distribution in the two eigen channels is passed back to the transmitter. Further, there is no loss in information if the power distribution vector P_1, P_2 is normalized to unity since the total power available (P) is known at the transmitter. Hence, we can design a computationally simple quantizer for the power allocation vector. Further, we have observed that a 2 bit quantizer effectively conveys all the information required for the power allocation at the transmitter. Additionally, we can gain up to one bit in feedback resources by noting that P_1 corresponding to the dominant eigenvector is always greater than or equal to P_2 corresponding to the other eigen channel. The quantizer for the power distribution vector is given in Table 1. Note that we set $P_2 = k P_1$, where $0 \le k \le 1$, with $P_1 \ge 0.5P$, where 2 bits are used to describe k.

Table 1: Quantizer used for the power allocation vector in the case of 4 transmit antennas and 2 receive antennas.

Water-filling solution	k
$0.75 \leq \frac{P_2}{P_1} \leq 1$	1
$0.5 \leq \frac{P_2}{P_1} \leq 0.75$	$\frac{1}{2}$
$0.25 \leq \frac{P_2}{P_1} \leq 0.5$	$\frac{1}{5}$
$0 \leq \frac{P_2}{P_1} \leq 0.25$	0

The effect of quantizing the power allocation vector to 2 bits as given in Table 1 can be seen in Figure 3 for the case of 4 transmit antennas and 2 receive antennas. We assume that the eigenvectors are known perfectly at the transmitter for this simulation to study the effects of quantization of the power allocation vector. Figure 3 shows the result of a two bit quantizer configured with perfect channel information. It can readily be seen that the two curves are essentially identical, so that the performance loss of the 2 bit power quantizer is negligible compared to the case of perfect channel state information.

We will now discuss the quantization of the two active eigenvectors of $H^{\dagger}H$. Consider a finite size beamformer codebook $\mathcal{C}$ of size N constructed as described in the previous section. We can first apply the quantization rule introduced in the last section to determine the best approximation to the dominant eigenvector among the

available vectors in $\mathcal{C}$. Note that the specification of this vector at the transmitter requires $\log_2(N)$ feedback bits. However, we can gain substantially in the specification of the second eigenvector by noting the following useful property.

Note that the eigenvectors of $H^{\dagger}H$ lie in $\mathbb{C}^t$. Further, the eigenvectors are all mutually orthogonal. Hence, the specification of the first eigenvector determines the subspace which contains the second active eigenvector. In particular, the second eigenvector lies in the t-1dimensional subspace which is orthogonal to the principal eigenvector. Hence, we can improve the description of the second vector significantly by constructing a second codebook in t-1 dimensions instead of the original t dimensional space.

However, it is not desirable to modify the composition of the codebook of the second eigenvector based on the first eigenvector, since the orthogonal subspace containing the second vector depends on the principal eigenvector. We therefore present an algorithm where both the beamformer codebooks are independent of the actual channel realization.

Let C_1 be a beamformer codebook in $\mathbb{C}^t$ comprising of $N_1=2^{B_1}$ vectors. Similarly, let C_2 be a beamformer codebook in $\mathbb{C}^{t-1}$ comprising of $N_2=2^{B_2}$ vectors. Let H be the channel realization, while V_1 and V_2 are the active eigenvectors of $H^{\dagger}H$.

We first quantize V_1 in C_1 using the quantization rule discussed in the last section. In particular, we pick $C_i^1 \in C_1$ (note that the superscript corresponds to the codebook index) such that $\|H(C_i^1)^{\dagger}\|_2$ is the maximum for all the vectors in C_1. Without loss of generality, we assume that C_1^1 maximizes the inner product with H among all the vectors in C_1.

Now, consider the vectors in C_2. We construct a codebook C_2' from C_2 such that C_2' lies in $\mathbb{C}^t$. Hence, C_2 is an embedding of C_2' in $\mathbb{C}^{t-1}$. By construction, C_2' is such that the first co-ordinate of all the vectors is set to zero. Hence, the vectors in C_2' lie

in the orthogonal subspace of the axis [1, 0, .., 0] of $\mathbb{C}^t$. Further, the embedding rule of C_2' into C_2 is that the first co-ordinate of C_2' is dropped to obtain the corresponding vector in C_2. Hence, if $C_i^{2'}=[0,c_1,c_2,..c_{t-1}]$, then the corresponding C_i^2 in C_2 is given by $[c_1,c_2,..c_{t-1}]$.

Now, we make use of the property that C_2' is in the orthogonal subspace of $e_1=[1,0,...0]$ in $\mathbb{C}^t$. In particular, we rotate the vectors in C_1 such that C_1^1 coincides with e_1. Let A be a t x t unitary matrix, constructed in a predetermined fashion from C_1^1 such that $AC_1^1 = e_1$ Now, we rotate the channel matrix H by the same matrix A before we quantize the second vector. Equivalently, we rotate the second vector V_2 by the matrix A to give $V_2' = AV_2$ Now, we quantize V_2' in the second beamformer codebook $C_2^{2'}$. Suppose $C_k^{2'}$ is the vector in $C_2^{2'}$ which maximizes the inner product with V_2'. Then, the transmitter gets the label k and the transmitter uses $A(C_2')^T$ for transmission, where the superscript T stands for matrix transpose operation. Note that A is a function of C_1^1 only and since the transmitter has information about C_1^1 via the feedback channel, the matrix A can be reproduced at the transmitter. Hence, both the resulting codebooks, C_1 and C_2 are independent of the actual channel realization.

Table 2: Table showing the decoding complexity as a function of the number of transmit antennas (t), receive antennas (r) and number of points in the modulation constellation (|Q|).

Transmission scheme	Decoding complexity
Space time coding	$\propto r\|Q\|^t$
Unit rank beamforming	$\propto r\|Q\|$
Spatial water-filling	$\propto r\|Q\|^{min(t,r)}$

Note that the quantized spatial water-filling solution requires joint coding and decoding across the active eigen channels. Hence, in the case of four transmit and two receive antennas which results in two active eigen channels, we will need joint coding across the two eigen channels. For instance, space time coding of rank 2 could be used to achieve the performance depicted in the next section. In the absence of channel state information, we would require a space time code of rank 4 corresponding to the four transmit antennas. Note that the decoding complexity of space time codes is exponential in the rank of the code. Hence, the quantized spatial water-filling solution results in significantly lower decoding complexity compared to the space-time coding, in addition to the benefits obtained in performance gains. The dependence of the decoding complexity on the number of transmit antennas and the number of receive antennas is shown in Table 2.

The performance of the quantized water-filling solution with 4 transmit antennas and 2 receive antennas is given in Figure 3. The figure shows the performance of a quantized water-filling scheme in comparison with the perfect information water-filling scheme, space time coding and perfect information eigenvector beamforming, all transmitting at the rate of R=6 bits/sec/Hz. For the quantized beamforming scheme, we have used 2 bits for spatial power control information, using the quantizer given in table 1. We observe the performances of two different codebook constructions in Figure 3. In the first case, the codebook C_1 was constructed in 4 dimensions with 16 vectors while the codebook C_2 was constructed in 3 dimensions with 16 vectors, thus requiring 3 feedback bits for each codebook. Hence, a total of 10 bits of feedback was used for this scheme. With 10 feedback bits, we note that we are just about 1 dB away from perfect feedback information while we obtain a gain of about 2 dB over space time coding. For the case of 8 feedback bits, we use 2 bits for spatial power control. Again, C_1 is in 4 dimensions, now with 8 vectors, while C_2 is in 3 dimensions with 8 vectors, thus requiring 3 bits each. With 8 feedback bits,

we observe a gain of about 1.5 dB over the space time coding scheme. It should also be pointed out that the unit rank beamforming scheme performs worse than the space time code in this case.

The beamforming schemes for multiple transmit and receive antenna systems presented above apply when only partial channel state information is available at the transmitter. The unit rank beamforming solution results in a low complexity decoding structures as well as performance gains over channel agnostic space time coding schemes. An algorithm for implementing higher rank transmission schemes, such as a spatial water-filling solution, using low complexity quantizers has also been illustrated. In all the cases, a few bits of channel state information at the transmitter can lead to substantial performance gains as well as reduction in decoding complexity.

In the next-section, we will show that the design of good beamformers for a multiple transmit and a single receiver system can be posed as the design of unitary space time constellations. The design of the unitary space time constellation is a dual problem of the design of good beamformers with the coherence time in the case of the unitary space time constellation given by the number of transmit antennas in the case of the beamformer design problem. Also, the number of transmit antennas in the equivalent unitary space time constellation design problem is set to unity (which is the number of receive antennas for the beamforming problem). We will establish the equivalence in the sequel and demonstrate the hitherto unknown use of unitary space time constellation as a beamformer with the help of an example. We will use the lower bound outage performance of beamformers for evaluating the performance of the unitary space time constellation as a beamformer.

Unitary space time constellations for multiple antenna systems were introduced by Marzetta and Hochwald in "Capacity of a mobile multiple-antenna communication

link in rayleigh flat fading", IEEE Transactions on Information Theory, pp 139 – 157, January, 1999. In particular, they showed that the unitary space time constellations achieve the capacity of multiple antenna systems when the channel information is not available at both the transmitter and the receiver. The design criterion for unitary space time constellations to minimize the probability of pair-wise error probability was given in "Unitary space-time modulation for multiple-antenna communications in rayleigh flat fading", IEEE Transactions on Information Theory, vol 46, pp. 543-564, March 2000. A unitary space time constellation consists of signals

$V_1, V_2, V_3, \ldots V_N$ where $V_i \in \mathbf{C}^{T \times M}$. Here T is the block length of the code (less than or equal to the coherence time of the channel) and M has the interpretation of number of transmit antennas. Also, $V_i^{\dagger} V_i = I$ for each i, for unitarity. It was shown that the design criterion for good unitary space time constellations is to minimize δ, where δ is defined as

$$\delta = \max_{1 \le l < l' \le N} \|\Phi_l^{\dagger} \Phi_{l'}\|, \tag{1}$$

where the norm used above is a scaled Frobenius norm of a matrix, the scaling factor being given by M in this case.

Now consider the design of a good beamformer comprising N vectors for a system with n transmit antennas and a single receive antenna. It can be shown that the design criterion for good beamformers is

$$\min_{C} \max_{1 \le l < l' \le N} |\langle C_l, C_{l'} \rangle|, \tag{2}$$

where $C_i \in \mathbf{C}^N$. The design criterion for unitary space time constellations given in (1) reduces to the design criterion of beamformers given in (2), if we set T=n and M=1 in (1). We see an equivalence between coherence time, in the unitary constellation design problem, and the number of transmit antennas, in the beamforming design problem.

Systematic unitary space time constellation as beamformer

The design criterion for beamformers which was introduced in the previous section is quite general and an exhaustive computer search can be used to construct a good beamformer containing a given number of beamforming vectors. We now look at a particular way of designing these beamformers, viz., using a Fourier based approach for unitary space time constellations. Hochwald et al. in "Systematic design of unitary space-time constellations" IEEE Transactions on Information Theory, vol. 46, pp. 1962-1973, September 2000, considered the problem of imposing structure on the unitary space time constellations for easier encoding/decoding as well as to reduce the dimensionality of the search space during the design process. The equivalence of good beamformers with good unitary space time constellations established in the previous section reveals the hitherto unknown properties of the systematic constructions. We will show that the unitary space time constellations designed by Hochwald et al. The cited reference serve as good beamformers for the dual problem with the new meaning of the number of transmit antennas attached to coherence time as discussed above.

Consider the set of linear block codes defined by the K x T generator matrix U, whose elements are in R_q, ring of integers modulo-q. The code $\mathcal{C}$ represented by U comprises codewords C_l given by $C_l = lU$, where l is a 1 x K vector with elements taken from R_q. Thus, the size of the codebook is given by $|\eth| = q^K$. The codewords are mapped into signals by mapping the integers in the codeword into components of a complex signal via the transformation

$$\phi(j) = \frac{1}{\sqrt{T}} e^{i\frac{2\pi}{q}j}, j = 0, 1, ..q - 1. \tag{3}$$

Finally, the unitary space time constellation is given by

$$\begin{aligned} \Phi_l &= \phi(C_l) \quad (4) \\ &= \frac{1}{\sqrt{T}} \begin{bmatrix} [e^{i\frac{2\pi}{q}[C_l]_1} \\ e^{i\frac{2\pi}{q}[C_l]_2} \\ \cdot \\ \cdot \\ e^{i\frac{2\pi}{q}[C_l]_T} \end{bmatrix}, 1 \leq l \leq N \end{aligned}$$

Suppose, we wish to design a beamformer comprising of N=16 vectors for n=8 transmit antennas transmitting at 2 bits/sec/Hz and a single receive antenna. Then the corresponding dual problem of designing unitary space time constellation reduces to designing a codebook of size N=16 with a single transmit antenna (i.e., M=1) and coherence time given by T=8. One of the codes designed for this problem in the cited reference is characterized by U=[1 0 3 14 15 11 10 8], K=1 and q=16.

The performance of this beamformer is given in Figure 5. The rate of transmission is R=2bits/sec/Hz. The corresponding performance predicted by the lower bound on outage is also given in the figure. It can be seen that the performance of the constructed beamformer comes very close (less than 0.05dB away) to the corresponding lower bound on outage probability. The performance of space time codes, which do not require any feedback information, is also given in the plots for reference. Note that there is gain of about 4dB with 4 bits of feedback compared to the space time codes. It should also be noted that there will be gains from reduced

receiver complexity also from the beamforming schemes which are not quantified in this document.

A similar construction taken from the cited reference again for n=8 transmit antennas and N=64 beamforming vectors is given in Figure 6. The unitary space time constellation in this case is characterized by N=64, T=8, K=3, q=4 and $U = [\ I_3 \ \ U']$, where I_3 is the 3x 3 identity matrix and U' is given by

$$U' = \begin{bmatrix} 2 & 3 & 3 & 3 & 0 \\ 2 & 0 & 3 & 1 & 1 \\ 0 & 3 & 2 & 3 & 3 \end{bmatrix} \tag{5}$$

For the case of 64 beamforming vectors, we present another straight-forward beamforming scheme for comparison. We use 1 bit to quantize the phase information of each one of the channel coefficients, leading to 8 bits in all, for 8 channel coefficients. The performance of this beamformer is also plotted in Figure 6. It can be seen that the performance of the constructed beamformer is again less than 0.05dB away from the corresponding bound . Further, the gain compared to the space time coding scheme now increases to about 6 dB. It can be observed from the plot there is lot of gain compared to the phase quantization scheme also. This shows that the systematic unitary space time constellations serve as very good beamformers. Some more constructions of the beamformers for n=8 transmit antennas can be obtained from the cited reference for the cases of N=133, 256, 529, 1296 and 2209 vectors.